MODELS CONSTRUCTED BY MR. C. F. PRENTICE FOR DEMONSTRATING REFRACTION BY CROSSED CYLINDERS.

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REFRACTION by crossed cylinders is a problem which it seems few, even of professed physicists, have cared to take hold of. Stokes' formulæ (the first we have any knowledge of), so far as published, are not applicable to all possible cases, and though he most probably worked out some general formulæ, they are not to be found in his printed works.

It has seemed to me, however, that for class demonstration and for those whose knowledge of optics is not sufficient to enable them to follow out an intricate mathematical demonstration, a model-representation was possible which would give a correct and intelligible idea of the course of rays after refraction by any two known cylinders with their axes crossed at any given angle. As cylinders act on light rays in one direction only, and that towards the focal line, it would be easy in two combined cylinders of known strength with their axes at a given angle, to find the final path of a ray that had been acted upon by both, according to the simple law of the parallelogram of forces. In accordance with this idea I constructed some rude models with wooden lenses and copper wires representing the light rays, which I forwarded to Mr. Charles F. Prentice of New York, with the request that he carry out and further elaborate the plan. The result has been the set of beautiful models that I have the pleasure of showing to you now. These show: 1st, refraction by a simple cross cylinder of 7 D.; 2d, refraction by 2 cylinders of 10 D. with an axial deviation of 90°; 3d, refraction by 2 cylinders of 10 D., axial deviation 45°; 4th, refraction by cylinders of 7 D. and 10 D., axial deviation 90°; 5th, refraction of cylinders of 7 D. and 10 D., axial deviation 45°. The lenses (of copper, gilded) have the same radii of curvature as the lenses they represent, and the direction of each ray after refraction is such as it would be in an actual lens of glass. On the focal planes (of gilded copper) are shown the amount and direction of deviation of each ray from its initial point of no-refraction, when acted upon by either lens separately, together with the parallelogram giving the extent and direction of deviation from the combined action of the two. On these models it is easy to follow any given ray after its refraction, and also to know the change in the form of a bundle of rays falling upon such a combination as well as the alteration in the position and direction of the focal lines with each change in the axial deviation of the two cylinders.

Mr. Prentice has, furthermore, at my suggestion, taken up the question mathematically and worked out, on the basis of this geometrical construction, formulæ which are found to be applicable to all possible cases of crossed cylinders. This work is found in the book[1] which I have the pleasure to show the members, and is to be commended on account of the thoroughness and simplicity with which the subject is treated. I think it can be said that there is no problem in connection with crossed cylinders which we are not now in a position to solve.

DISCUSSION.

DR. CARL KOLLER, New York.—I have always been much interested as to the best way of representing the rays after refraction by cylinders. Two or three years ago I observed some phenomena in the refraction of cylinders which had not been before observed. These phenomena were rather complicated and I tried to make them capable of being understood. I found that the simplest way was by the method of descriptive geometry. In this you have two or three planes at right angles to each other. In this way you can construct not only the focal lines but also the position and size of the images. The action of cylindrical lenses inclined to each other is also readily obtained.

[1] Dioptric formulæ for combined cylindrical lenses, applicable for all angular deviations of their axes, with 6 original diagrams and 1 photograph, by Chas. F. Prentice, pp. 48, James Prentice & Son, 178 Broadway, New York, 1888.

DR. EDWARD JACKSON, Philadelphia.—I think that in our *Transactions* of two years ago, the general formulæ for all possible combinations of cylindrical lenses will be found worked out. The adoption of wires in these models constitutes a great improvement. There is one thing that I have done with thread models which might be done with the wires, that is, to have the wires passing through one quadrant of one color while those that pass through other quadrants are of other colors. This makes the different relations of the rays more evident.

When Stokes devised his lens, I think there can be no doubt that he worked over the whole problem, but as he was only calling attention to that particular lens, he published nothing that did not relate directly to it.

The method of determining the resultant of two cylinders is extremely simple. Take for instance a cylinder of plus 2 D. combined with a cylinder of plus 1 D. at an angle of 30° between their axes. To determine graphically the equivalent it is only necessary to lay off a line representing 2 D. Then at an angle equal to twice the angle between the cylinders, making in this case 60°, lay off a line representing 1 D. and complete the parallelogram. The diagonal will then represent the cylindrical equivalent both in its cylindrical effect and the relative direction of the axis. If one cylinder is minus 1 D. instead of plus 1 D., the minus 1 D. is to be laid off in the opposite direction. In this way we can determine the cylindrical effect of a combination of any two cylinders at any angle between their axes with sufficient accuracy for practical purposes. If it is desired to determine it with greater accuracy the necessary formulæ will be found in the Transactions of two years ago (p. 273). The working out of these problems by descriptive geometry was pretty thoroughly done by Hooweg, who published his results in Græfe's Archives in 1873.

DR. CARL KOLLER.—I do not wish to say that I invented this method by descriptive geometry. I have quoted the one who first gave us the idea of treating these problems in that manner, that is Reūsch, but it had not been used in regard to the position and size of the images, etc. I have done this in a paper on the theory of cylindrical lenses.

DR. EDWARD JACKSON.—The phenomenon that Dr. Koller refers to is one with a part of which we are all familiar in testing cylindrical lenses to determine the axis. I remember his full explanation of it in Græfe's Archives.

DR. CARL KOLLER.—The phenomena which I observed were not known at that time, at least in Europe. Donders did not know them. When you look through the lens, holding it away from you, at an object and turn the lens the object rotates with twice the quickness of the lens. This is due to images which are not real but virtual. Every point of such an image consists of a real image point and a virtual image point.

DR. B. ALEX. RANDALL, Phila.—It may be of interest to allude to a very simple form of a model illustrating astigmatism. It is difficult to get students to understand this subject from diagrams, while it is easy with a model astigmatic eye. The model consists of an ordinary gas globe of clear glass—one having a *wide* opening below. On placing an oval of plane glass within one side of it, and filling the lenticular space so formed with fluid, it gives a good idea of astigmatism. When filled with water you have a focal length of about 5 D. in one and about 3 D. in the opposite plane. You get thus a hypermetropic astigmatism of about 2 D.—the axis of the globe being 33 C. M. By filling it with glycerine you get a myopic astigmatism. By filling the vitreous chamber with smoke you can demonstrate very beautifully the passage of rays by the use of the stenopaic slit. An astigmatic crystalline is also readily formed by placing together two of the convex ovals of thin glass, used in mounting colored photographs, and filling the interspace with fluid—and properly placed the correction of corneal by lenticular astigmatism is easily demonstrated.

DR. SWAN M. BURNETT, Washington.—The diagram that Dr. Jackson has placed on the board is exactly identical with that used by Stokes. Before the transactions of that year were published I had been working at the problem of a formula applicable to all cases. I found myself in difficulties, for I make no claim to be a mathematician. I therefore took the subject to Professor Harkness of the Naval Observatory. As soon as the papers of Drs. Jackson, Hay, and Oliver were published, I asked him to examine and see whether these formulæ were absolutely general. He said that there were certain cases to which he did not think these were applicable, and promised to work out some formula which would be applicable. He, however, did not do so. I had already constructed these models which I thought would certainly show the direction of any ray through any cylinders crossing at any angle. I sent them to Mr. Prentice, and he worked out the beautiful models you have before you. I have not had time to go over these formulæ of Mr. Prentice, but I think that if they are examined they will be found to be different from those previously published, and that they are applicable to all classes of cases. Mr.

Prentice has taken samples of all kinds and verified them practically with lenses according to the formulæ. What I wanted in the construction of the models was to show my students in a simple way the manner in which cylindrical lenses refract light. In regard to the suggestion of Dr. Jackson that the wires be colored, I think that it is a good one, but as they are, there is no difficulty in following out the course of the ray.

CASES OF OUTGROWTH ON THE OPTIC DISC.

DR. B. ALEX. RANDALL.

Having met little notice in ophthalmic literature of congenital outgrowths upon the optic disc, I have thought it perhaps worth while to report some of the cases which I have met where the anomaly seemed congenital, as being probably not at all rare and hence deserving notice. The first two are cases of tiny cyst. A case of a small cystic outgrowth was reported a couple of years ago by Dr. Kollock (Med. News, Oct. 23, '86), shortly after the observation and sketch of my first case, Fig. 1, Pl. I., and I have since seen another case illustrated in the second drawing, Fig. 2. In both of these cases the little pellucid vesicle stood out upon one of the retinal vessels, and was probably the cystic remains of a hyaloid artery; although the vessel apparently concerned is a vein in Fig. 1.

Three of the cases are of outgrowths of dense, white tissue upon the disc, but slightly prominent, and hiding, more or less completely, the underlying vessels. The last case, Fig. 6, shows a large, prominent outgrowth from the lower outer quadrant, extending down and out some five disc-diameters, and standing out some 6-8 D. into the vitreous. Here there is considerable evidence of pathological lesion.